BELLCOMM, INC.

955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

B69 11066

SUBJECT: Transfer Velocities Between Earth Space Stations and Satellites - Case 105-3

DATE: November 24, 1969

FROM: H. B. Bosch

ABSTRACT

In the calculation of impulsive velocity requirements for interorbital transfer it is generally insufficient to consider altitude and inclination differences only. The relative location of the ascending nodes of the two orbits is shown to be a dominant effect in the calculation of these velocities. As an extreme example, the transfer velocity between two polar orbits of low altitude can have a value between zero and 50,000 fps. Thus it is incorrect to consider two orbits of equal inclination as necessarily coplanar.

A second consideration is the differential rate of nodal regression between two orbits, due to earth oblateness. This causes the values of the required transfer velocities to oscillate with time, the frequency and amplitude of these oscillations depending on the orbit combination. A consequence of this is that a satellite which is not in essentially the same orbit as the space station will be accessible from the space station only at specified intervals.

To illustrate these two effects, circular orbits are chosen for three space stations and two classes of satellites. The space station orbits are 250 nm high at 55°, 75° and 90°. The satellite orbits are chosen as sun-synchronous (800 nm at 102°) - representative of meteorology and other earth application satellites - and low inclination (400 nm at 35°) - representative of some astronomy satellites.

Minimum ΔV requirements (those determined by considering inclination differences alone) are shown to be too restrictive. Higher velocities are shown to be more representative of interorbital maneuvers when the desired frequency and duration of [illegible] are considered.

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MEMORANDUM FOR FILE

Introduction

One factor which has received little attention in discussions of interorbital transfer velocities is the effect of the ascending nodes* of the two orbits. As an example, the transfer velocity between two polar, low altitude orbits could have any value between zero and 50,000 fps, depending on the relative orientation of the two orbital planes.

It is thus not sufficient to consider inclination differences alone (see Reference 1, section III.2). The actual transfer angle (α) between two orbits of inclination i_o and i, and whose ascending nodes are separated by an angle $\Delta\Omega$, is given by

$$\cos \alpha = \cos i_o \cos i + \sin i_o \sin i \cos \Delta\Omega$$

as is illustrated on Figure 1.

Figure 2 shows examples of two-burn velocities for noncoplanar transfer** to orbits of low to medium altitude from space station orbits at 55° and 90°. These curves show that altitude change consumes a very small portion of the total energy required, whereas inclination change and node separation are major effects of equal importance. Note that

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*The ascending node is that point on the equatorial plane where the satellite crosses from the southern to the northern hemisphere.

**A Hohmann transfer between two circular orbits where the plane change is accomplished by adding an appropriate out-of-plane component to the apogee impulse (for details see Appendix and Figure 6 of Reference 2).

transfering between two 55° orbits whose ascending nodes are separated by only 40° requires as much propellant as transfering between a 55° and a 23° orbit with coincident nodes.

Effect of Nodal Regression

The node separation between two orbits generally changes with time. The nonsphericity of the earth essentially causes the ascending node of an orbit to move at a rate depending on the size, shape and inclination of the orbit (see Reference 3). For circular orbits of radius r and inclination i this rate ($\dot{\Omega}$) may be calculated from

$$\dot{\Omega} = - \frac{3}{2} J_2 \sqrt{\mu/R^3} (R/r)^{7/2} \cos i$$

where $J_2 = 1.0823 \times 10^{-3}$ (an oblateness parameter for earth)
$\mu = 1.40766 \times 10^{16}$ ft^3/sec^2 (gravitation parameter for earth)
$R = 2.0925 \times 10^7$ ft = 3442 nm (mean radius of earth).

This relationship is shown on Figure 3 for orbits of various altitude h. The nodes of orbits of inclination less than 90° move westward (regression) and those of inclination greater than 90° move eastward (progression). Polar orbits have essentially stationary nodes.

It is apparent that orbits of different altitude and inclination will have different regression rates and thus there will be a relative motion between their ascending nodes. This will make the transfer velocities between two orbits vary with time.

Time Variation of Velocity Requirements

For illustrative purposes three space station orbits of 250 nm altitude were chosen, at inclinations of 55°, 75° and 90°. One way transfer velocities from these to a sun-synchronous orbit (800 nm, 102°) and to a low inclination orbit (400 nm, 35°) are shown on Figures 4 and 5. The sun-synchronous orbit is typical of meteorology and other earth applications satellites, and the low inclination orbit represents some astronomy satellites. The reasons for choosing such orbits for these applications are discussed in the Appendix of Reference 1.

The transfer velocity between a polar and a sun-synchronous orbit can vary between 6,000 fps and 47,000 fps (Figure 4) depending on the relative position of the nodes. Since a sun-synchronous orbit (by definition) progresses at one degree per day, and a polar orbit is stationary, the ascending nodes of the two orbits will coincide once a year, thus presenting a minimum-ΔV opportunity only once a year. On the other hand a 250 nm, 55° orbit regresses at 4.5 degrees per day (Figure 3) presenting between five and six minimum-ΔV opportunities each year for visiting a sun-synchronous satellite. Such relationships between minimum velocity and frequency of opportunities have been synopsized on Figure 6 for the six cases considered.

Some Observations

It is apparent from Figures 4 and 5 that, depending on the length of stay in the visited orbit, the return velocity requirement will generally be different from the out-going velocity requirement. For example, transfering to a sun-synchronous orbit from a 55° space station (Figure 4) at a minimum-ΔV opportunity requires 19,200 fps. Returning ten days later requires 27,200 fps, making a roundtrip requirement of 46,400 fps. However, leaving five days before and returning five days after a minimum-ΔV opportunity still allows ten days staytime but the roundtrip requirement is now 43,200 fps. Similar trade-offs between oneway ΔV requirements and desired duration of opportunity are shown on Figure 7.

Although the orbit regression due to oblateness causes transfer velocity requirements to oscillate between the extremes shown on Figures 4 and 5, it is also this same oscillation which periodically brings about a minimum-ΔV opportunity. With planning, therefore, this differential regression rate can be quite advantageous.

Finally, it should be noted that phasing maneuvers upon arrival at the satellite orbit or upon return into the space station orbit are not considered here. The velocity calculations are based on the assumption that the target vehicle is "there" upon arrival. For example, it is shown in Reference 2 that phasing in geosynchronous orbit can require up to 4,500 fps. Similarly, phasing in a low altitude circular orbit can require up to 11,500 fps. These values correspond to the extreme case of in-orbit phasing of 180° with a single revolution on a phasing ellipse, and later returning to the original position in the orbit. By using phasing ellipses for more than one revolution these velocity requirements can be reduced, as is further shown in Reference 2.

Since circular orbits of different altitude have different periods, phasing could also be accomplished by waiting an appropriate length of time in the initial orbit before departing on the transfer ellipse. Such waiting, however, could also increase the plane change requirements as is shown in this memorandum. Therefore phasing has to be considered for specific missions and the additional velocity requirements can be obtained from Reference 2.

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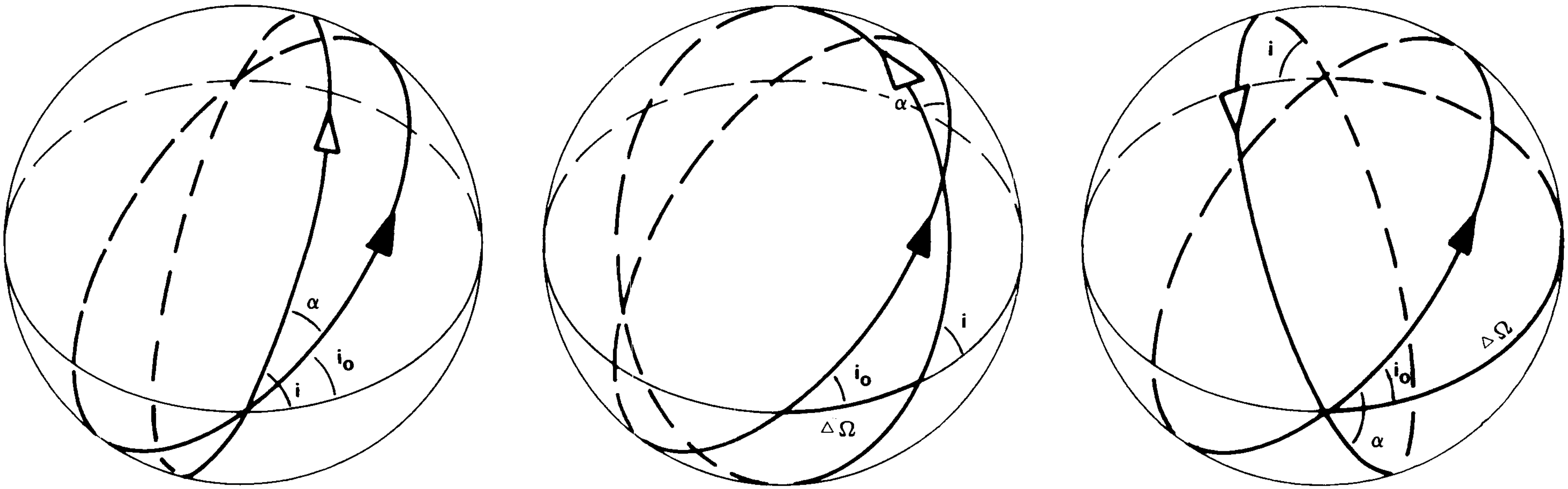
References
Figures 1-7

REFERENCES

1. Bosch, H. B., "Preliminary Survey of the Potential for Satellite Servicing," Bellcomm Memorandum for File, 22 May 1969.

2. Bosch, H. B., "Characteristic Velocity Requirements for Intraorbital Phasing and Interorbital Transfer Missions," Bellcomm Memorandum for File, 14 July 1969.

3. Wolverton, R. W., ed., Flight Performance Handbook for Orbital Operations, John Wiley & Sons, Inc., 1963, p. 2-228 ff.

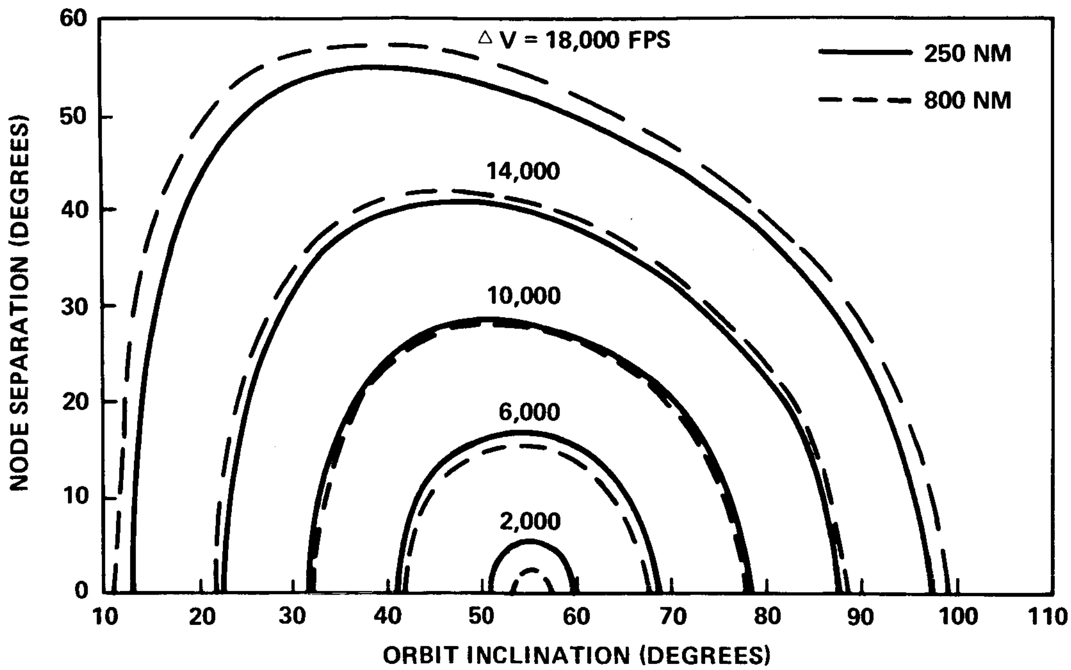


(a) $\Delta\Omega = o$, $\alpha = i - i_o$

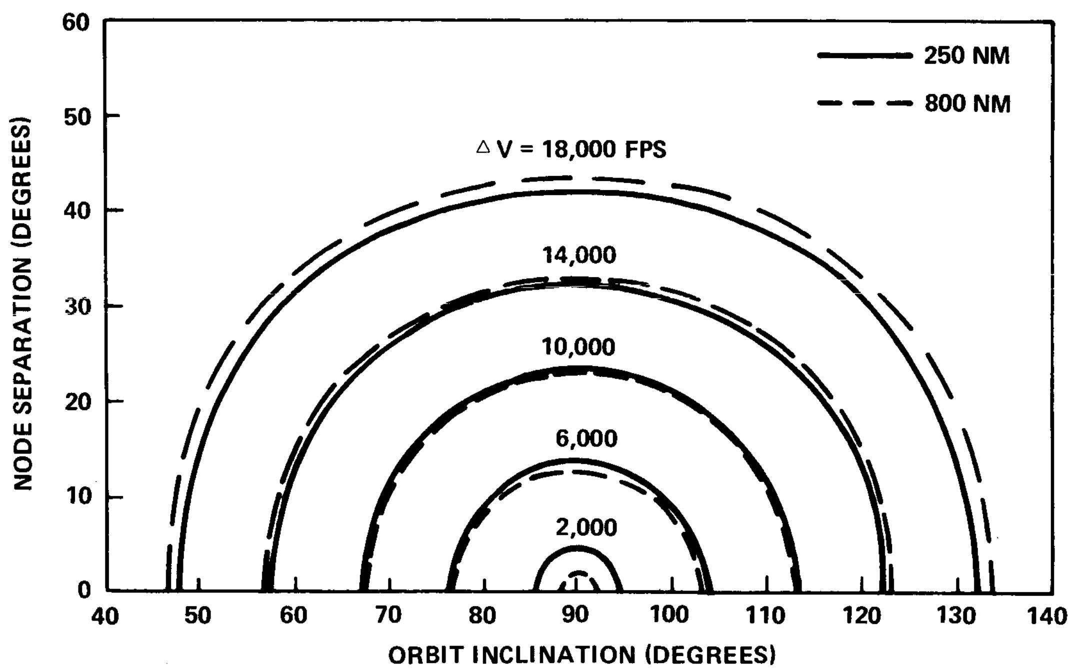
(b) $o < \Delta\Omega < 180°$, $i - i_o < \alpha < i + i_o$

(c) $\Delta\Omega = 180°$, $\alpha = i + i_o$

FIGURE 1. EFFECT OF THE SEPARATION OF ASCENDING NODES ($\Delta\Omega$) ON THE ACTUAL PLANE CHANGE (α) BETWEEN TWO ORBITS OF INCLINATION i_o AND i.



(A) FROM 250 NM, 55° SPACE STATION ORBIT



(B) FROM 250 NM POLAR SPACE STATION ORBIT

FIGURE 2. CONTOURS OF CONSTANT ONEWAY ΔV, INDICATING CIRCULAR ORBITS AT 250NM AND 800 NM WHICH ARE ACCESSIBLE FROM TWO SPACE STATION ORBITS BY TWO-BURN TRANSFER MANEUVERS

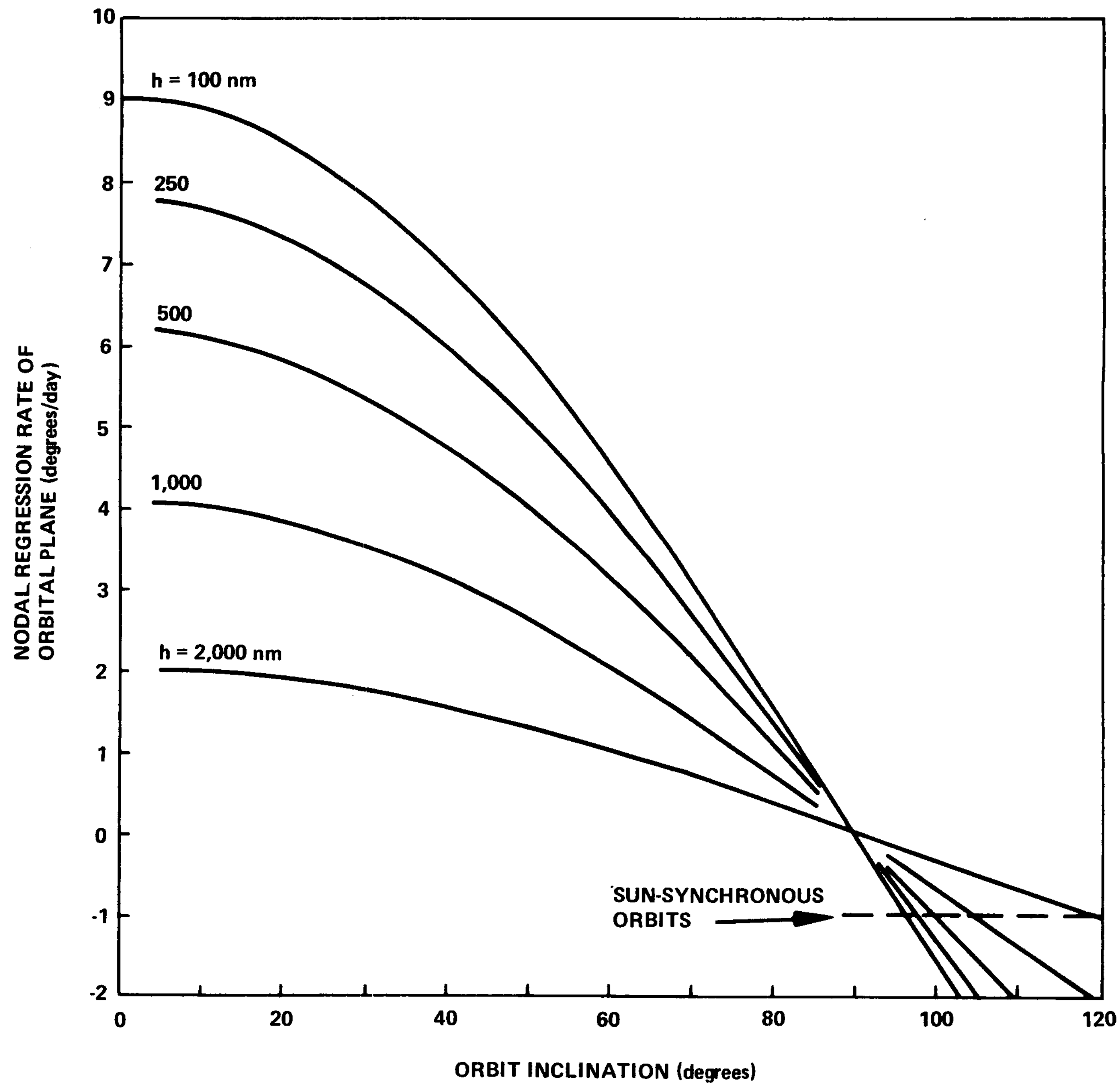


FIGURE 3. ALTITUDE AND INCLINATION EFFECTS ON NODAL REGRESSION RATE OF CIRCULAR ORBITS

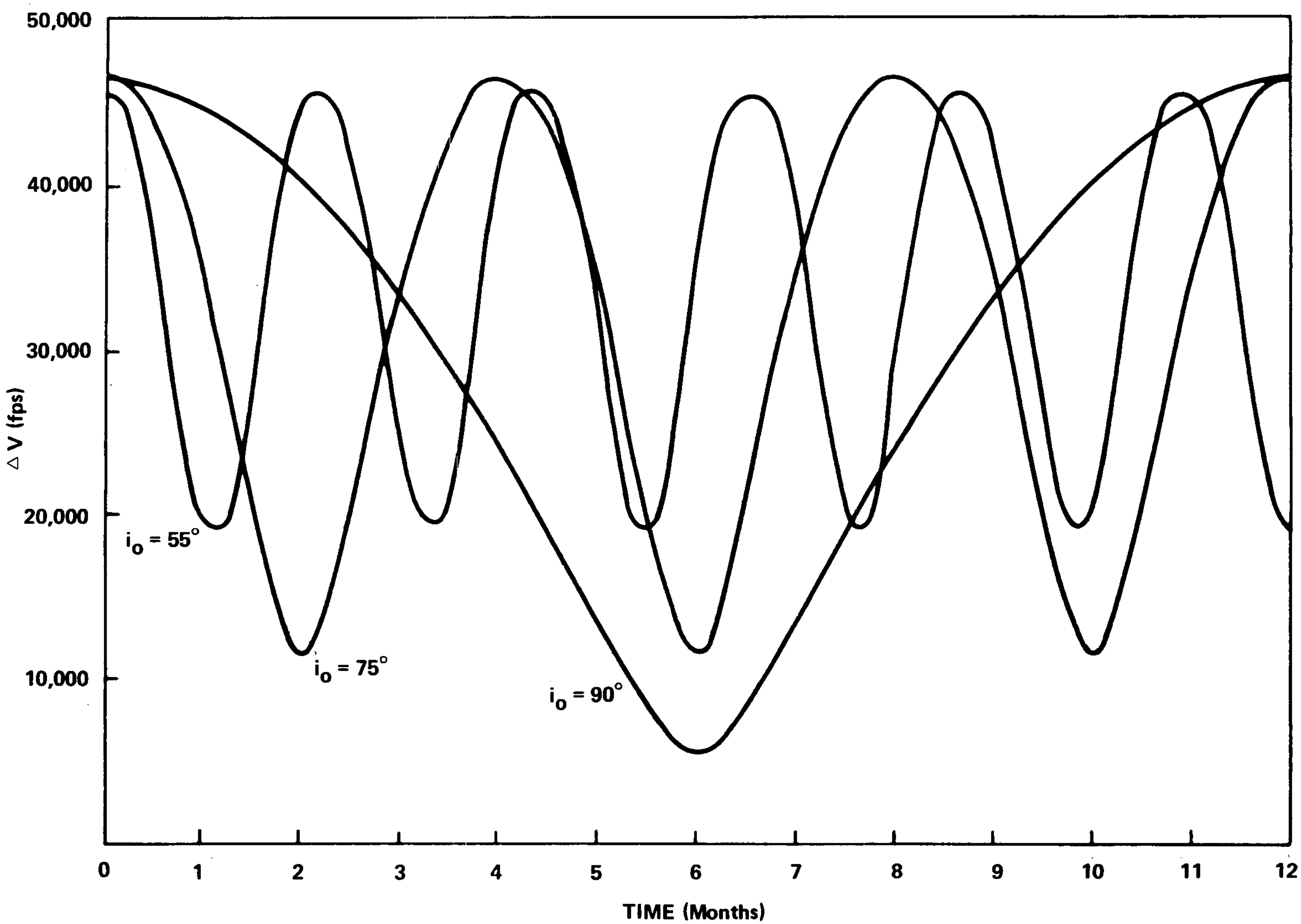


FIGURE 4. TIME VARIATION OF ΔV REQUIREMENT FOR TRANSFER TO A TYPICAL SUN-SYNCHRONOUS (i = 102°, h = 800 nm) ORBIT FROM A 250 nm SPACE STATION ORBIT AT INCLINATION i_o.

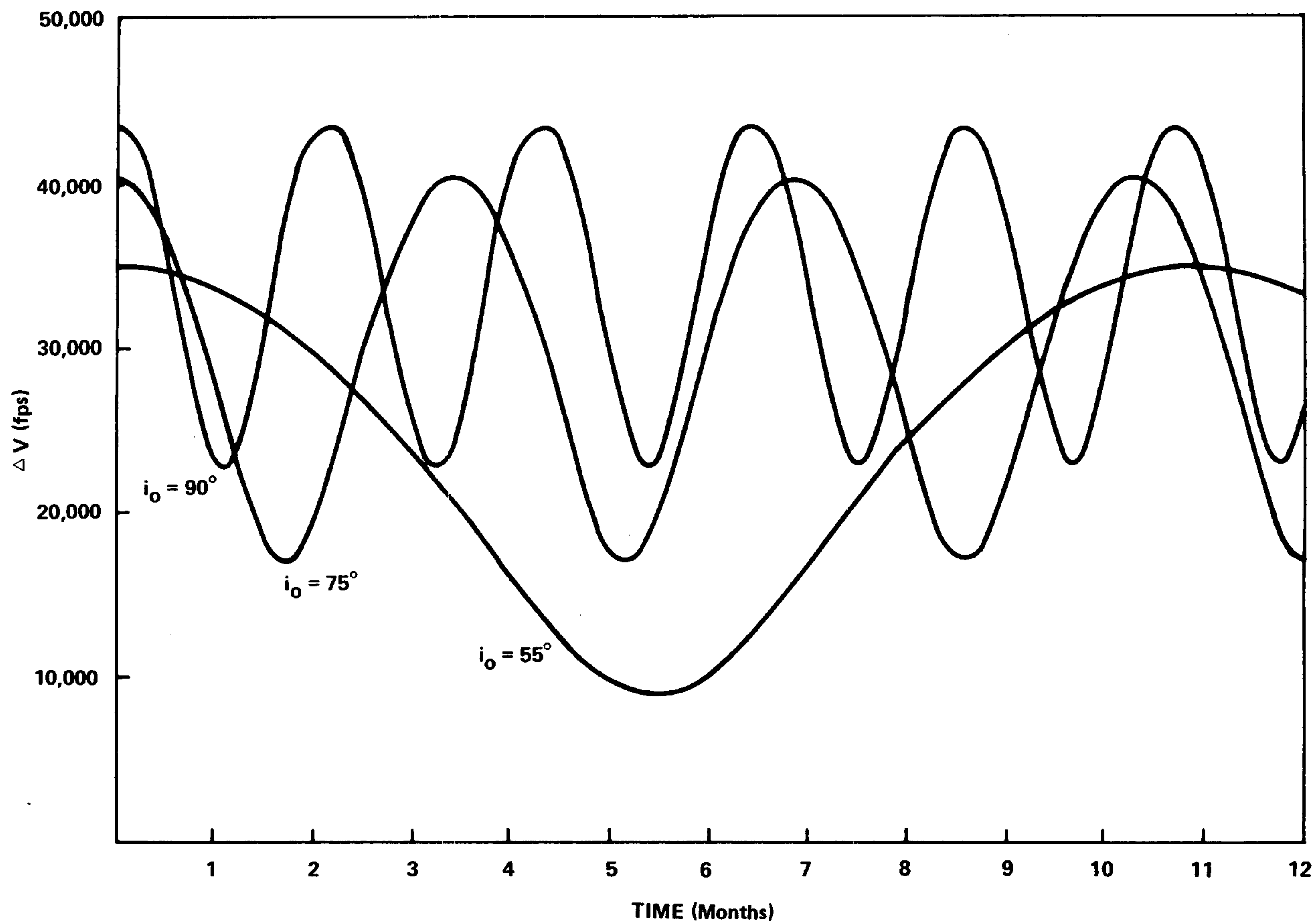
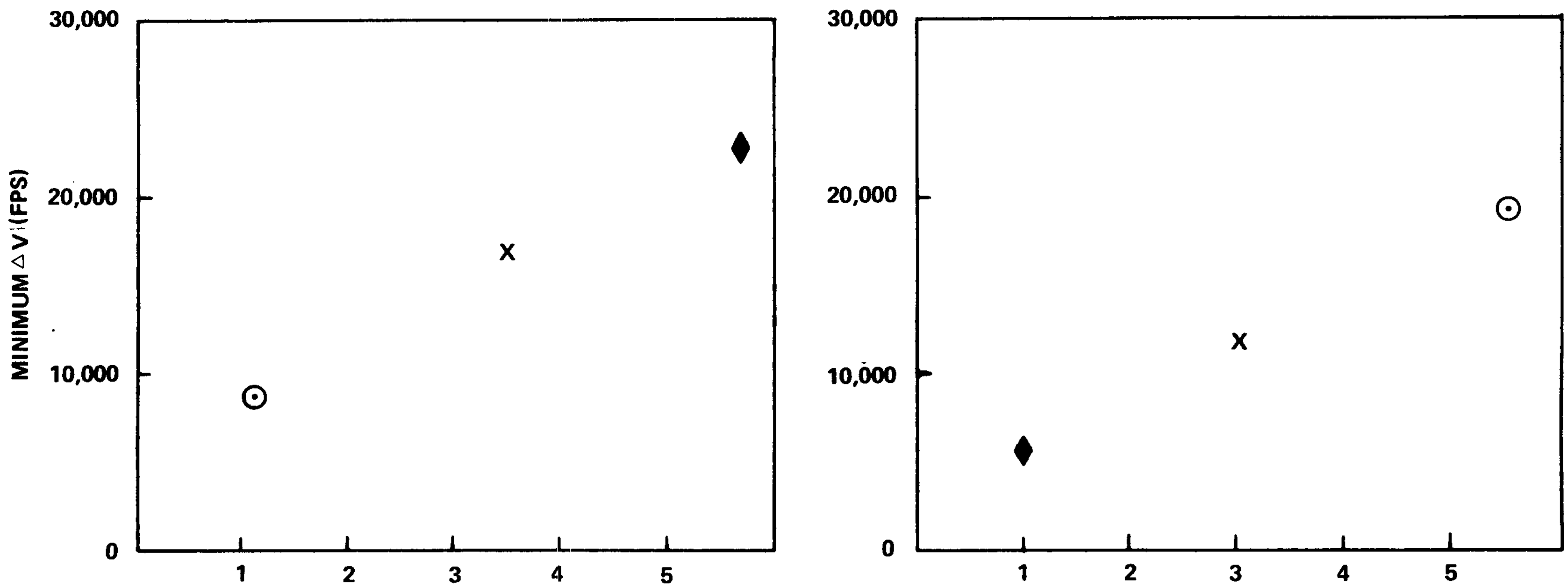


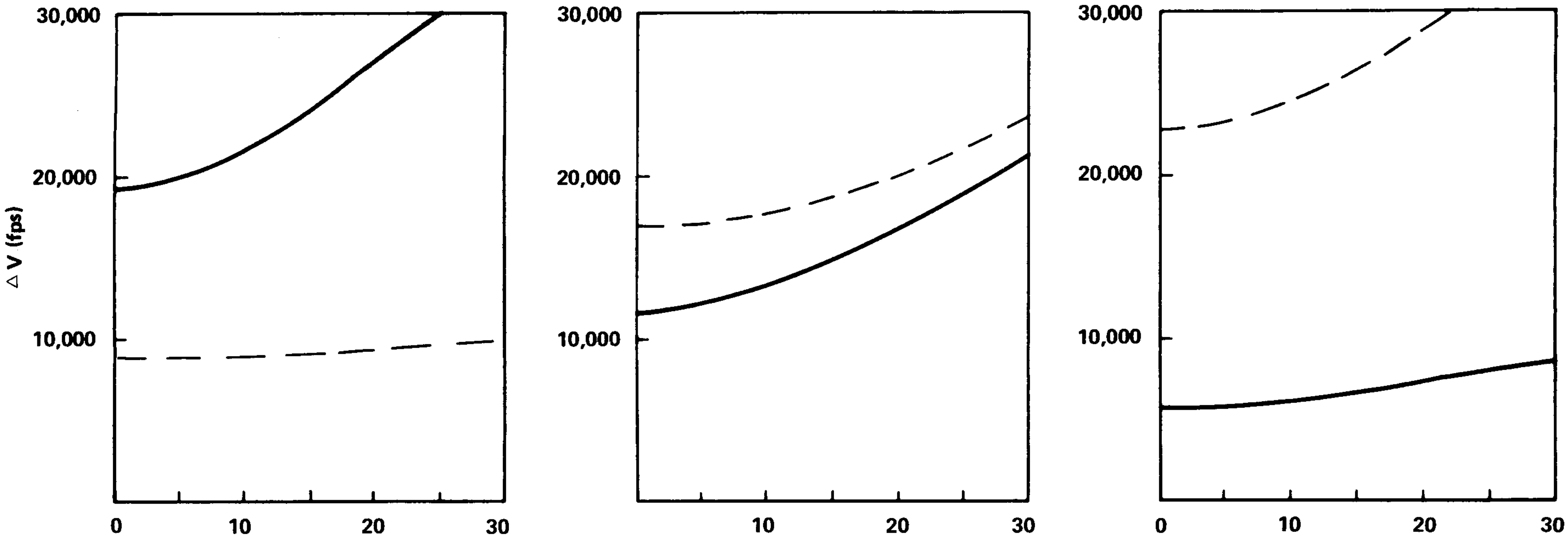
FIGURE 5. TIME VARIATION OF ΔV REQUIREMENT FOR TRANSFER TO A TYPICAL LOW INCLINATION ($i = 35°$, $h = 400$ nm) ORBIT FROM A 250 nm SPACE STATION ORBIT AT INCLINATION i_o.



(a) NUMBER OF OPPORTUNITIES PER YEAR TO VISIT SATELLITE AT 35°, 400 nm.

(b) NUMBER OF OPPORTUNITIES PER YEAR TO VISIT SATELLITE AT 102°, 800 nm.

FIGURE 6. NUMBER OF MINIMUM–ΔV OPPORTUNITIES TO VISIT TYPICAL LOW INCLINATION AND SUN-SYNCHRONOUS SATELLITE ORBITS FROM 250 nm SPACE STATION ORBITS AT 55° (⊙), 75° (X) AND 90° (♦).



(a) OPPORTUNITY DURATION (days) FROM SPACE STATION AT $i_o = 55°$, $h_o = 250$ nm.

(b) OPPORTUNITY DURATION (days) FROM SPACE STATION AT $i_o = 75°$, $h_o = 250$ nm.

(c) OPPORTUNITY DURATION (days) FROM SPACE STATION AT $i_o = 90°$, $h_o = 250$ nm.

FIGURE 7. ONEWAY ΔV REQUIREMENT FOR VISITING SATELLITES IN TYPICAL LOW INCLINATION (––––) AND SUN-SYNCHRONOUS (——) ORBITS AS A FUNCTION OF DESIRED OPPORTUNITY DURATION.